

Couplings of $NN(n\pi)$ $n \geq 1$

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Abstract

Determinations of the couplings of $NN(n\pi)(n \geq 1)$ are reported. The study is based on both a quark model of nucleon and a chiral field theory of mesons. The coupling of $NN\pi$ is predicted and is in agreement with current value. It shows that the coupling $NN2\pi$ is resulted in the nature that pion is a Goldstone boson. The couplings of $NN(n\pi)(n \geq 2)$ are predicted by this approach.

It is known that the $NN\pi$ vertex is an important coupling of the strong interactions. This coupling is a problem of nonperturbative QCD. There are already extensive studies on this coupling by various approaches. The value of the coupling constant of nucleon and pion has been determined [1]. Besides the $NN\pi$ coupling the $NN2\pi$ vertices have been introduced to study the $\pi - N$ s-wave interaction in Ref. [2] and these 2π couplings [2] have been used to study the N - N potential in Ref. [3]. In Refs. [4] the current algebra and the PCAC have been applied to calculate the matrix elements in which additional soft pions are emitted or absorbed.

In this paper a quark model of nucleon [5] and a chiral field theory of mesons [6] are employed to determine the N- π coupling and to explore the nature of chiral symmetry of the couplings of N- 2π and $NN(n\pi)(n \geq 3)$.

In Ref.[5] we have proposed an approach of studying the electromagnetic and the weak interactions of nucleon (baryon) in 70's. In this approach SU(6) symmetry is applied to construct the quark wave functions of baryons in the rest frame and the transition matrix elements. The effective electromagnetic and the charged weak currents of quarks are constructed and applied to study the electromagnetic and the charged weak processes of baryons. In this approach the $G_M^p(q^2)$, a parameter from the model, and the $G^A(0)$ are taken as inputs. In 1975 this model predicted that in small range of q^2 the ratio $\mu_p G_E^p(q^2)/G_M^p(q^2)$ is about $\simeq 1$ and decreases with q^2 when $q^2 > 1 \text{ GeV}^2$. These predictions agree with the new data.

This model also predicts a very small $G_E^n(q^2)$ and $G_M^n = -\frac{2}{3}G_M^p$. The $\mu_{p \rightarrow \Delta} = 1.23 \frac{2\sqrt{2}}{3}\mu_p$, the helicity amplitudes of $\gamma p \rightarrow \Delta$, and very small negative S_{1+} , E_{1+} are predicted too. By the way in this model the shape of a proton is spherical in the rest frame. In the moving frame the shape of the proton is no longer spherical because of Lorentz contraction. For weak interactions of the charged currents the axial-vector form factor $G^A(q^2)/G_A(0)$ is predicted and in very good agreement with the data. The predicted cross sections of the scattering of the neutrino (antineutrino) and the nucleon by the charged weak currents are in good agreements with data too.

This quark model of nucleon is applied to study the strong interactions of nucleon - pions in this paper. It is very important to notice that the pion is a Goldstone boson and it is massless in the chiral limit. The couplings between quarks and pions are obtained from a chiral field theory of mesons that we have proposed in Ref. [6]. In this chiral field theory the pion is a Goldstone boson and the mass of the pion is resulted in explicit chiral symmetry breaking by the current quark mass. The pion mass [6,7], the pion form factor [8], $\pi - \pi$ scattering [6], and the anomaly $\pi^0 \rightarrow \gamma\gamma$ [6,9] etc. are studied and the theory agrees with data well. The physics processes of the pseudoscalar, the vector, and the axial-vector mesons are studied too. Theory is in good agreement with data.

The nucleon model and the chiral field theory of mesons are applied to study the strong interactions of nucleon and pions in this paper. For convenience the Lagrangian of the chiral

field theory of mesons in the case of two flavors Eq. (1) of Ref. [6] is copied as following

$$\begin{aligned}\mathcal{L} = & \bar{\psi}(x)(i\gamma \cdot \partial + \gamma \cdot v + \gamma \cdot a\gamma_5 - mu(x))\psi(x) \\ & + \frac{1}{2}m_0^2(\rho_i^\mu \rho_{\mu i} + \omega^\mu \omega_\mu + a_i^\mu a_{\mu i} + f^\mu f_\mu),\end{aligned}\tag{1}$$

where ψ is the quark fields, $a_\mu = \tau_i a_\mu^i + f_\mu$, $v_\mu = \tau_i \rho_\mu^i + \omega_\mu$, $u = e^{i\gamma_5(\tau_i \pi_i + \eta)}$, the quantity m is originated in the quark condensate, and the m_0 is related to the mass of the ρ meson [6].

The kinetic terms of the meson fields are generated from the quark loop diagrams and the physical meson fields are defined [6]. For example

$$\pi^i \rightarrow \frac{2}{f_\pi} \pi^i \text{ (physical)}$$

where the pion decay constant f_π is determined by the kinetic term of the pion field and $f_\pi = 0.186$ GeV is taken. Another parameter g is generated from the kinetic term of the ρ field and $g = 0.395$ is determined by input the decay rate of $\rho \rightarrow ee^+$. The m is determined by f_π and g to be $m = 0.242$ GeV.

There are two sources of the pion field: the non-linear σ term in Eq. (1) is the first source and the mixing between the pion and the axial-vector fields [6] generates another pion field

$$-\frac{c}{g} \frac{2}{f_\pi} \partial_\mu \pi^i,$$

where $\frac{c}{g} = \frac{f_\pi^2}{2g^2 m_\rho^2}$. The couplings between the quark fields and the pion fields is found from Eq. (1)

$$\mathcal{L} = -\bar{\psi} \left\{ \frac{2im}{f_\pi} \gamma_5 \tau^i \pi^i + \frac{c}{g} \frac{2}{f_\pi} \gamma_\mu \gamma_5 \tau^i \partial_\mu \pi^i \right\} \psi.\tag{2}$$

In the model of nucleon [5] the effective EM current of the quarks has a term of anomalous magnetic moment of the quark and the effective charged weak current of the quarks is expressed as

$$\lambda\gamma_\mu\gamma_5$$

where the parameter λ is determined as

$$\lambda = \frac{5}{3}G_A$$

which is the same as the one obtained in the model of SU(6) symmetry. In Eq. (2) the axial-vector part $\tau^i\gamma_\mu\gamma_5$ is the same as the charged weak current. Therefore, the effective Lagrangian of Eq. (2) should be rewritten as

$$\mathcal{L} = -\bar{\psi}\left\{\frac{2im}{f_\pi}\gamma_5\tau^i\pi^i + \frac{c}{g}\frac{2}{f_\pi}\lambda\gamma_\mu\gamma_5\tau^i\partial_\mu\pi^i\right\}\psi. \quad (3)$$

Eq. (3) is used to calculate the transit matrix element of $N \rightarrow N + \pi$ and to determine the coupling of $i\bar{N}\gamma_5\tau^iN\pi^i$. Before the calculation it is worth to mention that this theory has both the features of the current algebra and the non-perturbative QCD. In Refs. [4] the current algebra and PCAC have been used to study multi-pion production and absorption. Based on the vector currents, the axial-vector currents, and the non-linear σ model of the current algebra the vector mesons, the axial-vector mesons and the pseudoscalar mesons are introduced to this Lagrangian (1) which is $U(2)_L \times U(2)_R$ global chiral symmetric. Some of the results of current algebra are revealed from this theory [6], for example, the scattering

lengths and the slopes of the $\pi-\pi$ scattering and the two Weinberg sum rules [10]. The PCAC is satisfied [11]. On the other hand, this chiral field theory has some of the major features of nonperturbative QCD: 1) the quark loop diagrams are at the leading order in the N_C expansion and the meson loop diagrams are at higher orders; 2) this theory has the explicit chiral symmetry in the limit $m_{light\ quarks} \rightarrow 0$ and it is broken explicitly by the current quark masses; 3) there is dynamical chiral symmetry broken by the quark condensation which plays a very important role in this theory.

Replacing the EM currents or the charged weak currents in the transit matrix elements of the nucleon Eqs. (49, 197) in Ref. [5R] by the vertex of quark - pion (3), the πN coupling is obtained by calculating the transit matrix element $N \rightarrow N + \pi$

$$\begin{aligned}
\langle B_\lambda(p')_{U'} \pi(q) | S | B_\lambda(p)_U \rangle &= i(2\pi)^4 \delta(p - q - p') \frac{1}{2} \int dx'_1 dx'_2 dx_1 dx_2 \\
&\quad \bar{B}_{\alpha\beta\gamma,ijk_1}^{\lambda'}(x'_1, x'_2, 0)_{U'} (\tau^i)_{k_1 k'_1} \left\{ -\frac{2im}{f_\pi} \gamma_5 \pi^i - \frac{c}{g} \frac{2}{f_\pi} \lambda \gamma_\mu \gamma_5 \partial_\mu \pi^i \right\} \gamma_\gamma \gamma' \\
&\quad M(x'_1, x'_2, x_1, x_2) B_{k'_1 j i, \gamma' \beta \alpha}^\lambda(0, x_2, x_1)_U,
\end{aligned} \tag{4}$$

where the B with indices are the nucleon wave functions Eqs. (33, 46) in Ref. [5R], U' and U are the flavor states of the final and initial baryons [5] respectively, the function $M(x'_1, x'_2, x_1, x_2)$ defined in Ref. [5] is the effect of other two quark lines in the transit matrix element and by the requirement of the SU(6) symmetry it is simply assumed that $M(x'_1, x'_2, x_1, x_2)$ is a scalar function and it is unknown, which is absorbed by the form

factors defined in Ref. [5]. It is assumed that in the rest frame the wave functions contain s-wave only and they observe SU(6) symmetry. Using Lorentz transformation the wave functions are boosted to moving frame. The details can be found in Ref. [5]. There are two Lorentz invariant spacial functions $f_{1,2}(x_1, x_2, x_3)$ in the quark wave functions of nucleon and the assumption $f_2 = af_1$ is made [5] and $a = 4.51$ is determined. It is shown that this assumption works well in the range of $q^2 < 5 \text{ GeV}^2$.

Calculating the matrix element (4), it is obtained

$$\mathcal{L}_{NN\pi} = g_{NN\pi} F_{NN\pi}(q^2) i \bar{N} \gamma_5 \tau^i N \pi^i, \quad (5)$$

$$g_{NN\pi} = \frac{2m_N G_A}{f_\pi} \left\{ \frac{m}{m_N G_A} \left(\frac{1}{a} + \frac{5}{3}a - 1 \right) - \frac{2c}{g} \right\}, \quad (6)$$

$$F_{NN\pi}(q^2) = D_2(q^2) \left\{ 1 - \frac{1}{12f_\pi} \frac{1}{g_{NN\pi}} \left\{ 10ma - \frac{2c}{g} \lambda(6a+4)m_N \right\} \frac{q^2}{m_N^2} \right\}, \quad (7)$$

where q is the momentum of the pion, $q^2 = (p - p')^2$, and

$$D_2(q^2) = \frac{1}{\mu_p} G_M^p(q^2) \frac{1}{1 + 2.39 \frac{q^2}{4m_N^2}} = \frac{1}{(1 + \frac{q^2}{0.71})^2 (1 + 2.39 \frac{q^2}{4m_N^2})}$$

Eq. (112) in Ref. [5R] determined by fitting the magnetic form factor of the proton.

The coupling constant $g_{NN\pi}$ is defined at $q^2 = 0$ of the form factor of the $NN\pi$ vertex. If taking $q^2 = m_\pi^2$, there is a very small correction from Eq. (6). The m_N and the G_A are known and all other the parameters in Eqs. (6,7) are determined in Ref. [5,6]. It obtains

$$\frac{g_{NN\pi}^2}{4\pi} = 13.05. \quad (8)$$

This value is consistent with the values of the $g_{NN\pi}$ presented in Ref. [1]. It is worth to point out that because

$$\frac{m}{m_N G_A} \left\{ \frac{1}{a} + \frac{5}{3}a - 1 \right\} - \frac{2c}{g} = 1.01$$

the Goldberg-Treiman relation is satisfied pretty well in this approach. The satisfaction of the Goldberg-Treiman relation means the parameters determined from the EM and the weak interactions of the nucleon in Ref. [5] and the meson theory [6] are reasonable. Eq. (6) shows that there is cancellation between both the pseudoscalar and the axial-vector parts of the coupling constant $g_{NN\pi}$. Inputting the values of the parameters the form factor $F_{NN\pi}$ is determined as

$$F_{NN\pi}(q^2) = D_2(q^2) \left\{ 1 - 0.096 \frac{q^2}{m_N^2} \right\}. \quad (9)$$

The radius of this form factor is revealed from Eq. (9)

$$\langle r \rangle = 0.916 \text{ fm}. \quad (10)$$

Eq. (9) shows that the form factor of the nucleon-pion decreases faster with q^2 than $\frac{1}{\mu_p} G_M^p(q^2)$ whose radius is 0.81 fm . The charged axial-vector form factor of nucleon $\frac{1}{G_A(0)} G_A(q^2) = D_2(q^2) (1 + 1.125 \frac{q^2}{M_N^2})$ Eq. (218) in Ref. [5R] agrees with data well and its radius is 0.72 fm . $F_{NN\pi}(q^2)$ decreases faster than $\frac{1}{G_A(0)} G_A(q^2)$ too.

In Refs.[2,3] the vertices between nucleon and two pions are introduced. Besides the coupling $NN\pi$ why it is needed to introduce the vertex $NN\pi\pi$? In this approach this

question is answered. In the chiral limit $m_q \rightarrow 0$ Lagrangian (1) is $U(2)_L \times U(2)_R$ chiral symmetric and the pion is massless. By adding the quark mass matrix $-\bar{\psi}M\psi$ to the Lagrangian (1) [6,7], where M is the mass matrix of the u and d quarks, the chiral symmetry is explicitly broken by current quark mass. In Eq. (1) the u matrix is via the nonlinear σ model of the current algebra introduced to make the Lagrangian (1) chiral symmetric. If taking the pion field into account only

$$u = e^{i\gamma_5\tau^i\pi^i} = 1 + i\gamma_5\tau^i\pi^i - \frac{1}{2}\pi^i\pi^i + \dots \quad (11)$$

Eq. (11) shows that besides the coupling of quarks and one pion there are coupling between the quarks and two pions

$$-im\bar{\psi}\gamma_5\tau^i\psi\pi^i + \frac{m}{2}\bar{\psi}\psi\pi^2. \quad (12)$$

Using Eq. (12), the pion mass in the order of $O(m_q)$ is revealed from the calculations of two quark loops [6,7]

$$m_\pi^2 = -\frac{1}{3}\frac{4}{f_\pi^2} \langle \bar{\psi}\psi \rangle (m_u + m_d). \quad (13)$$

where the quark condensate is from the m (1). This is the Gell-Mann, Oaks and Renner formula [12]. Eq. (13) shows that in the chiral limit, $m_q = 0$, the pion is massless and a Goldstone meson. After adding the quark masses, there are two mass dimensions m and the current quark mass. Why the $m_\pi^2 \propto m_u + m_d$ not $m_\pi^2 \propto m$? The reason is that there are cancellations between the two quark loops of the two vertices (12). If only one of the two

terms of Eq. (12) is taken into account $m_\pi^2 \propto m$ is revealed. Therefore, besides the coupling of

$$-im\bar{\psi}\gamma_5\tau^i\psi\pi^i$$

the vertex of two π

$$\frac{m}{2}\bar{\psi}\psi\pi^2 \quad (14)$$

is necessary for pion to be a Goldstone boson. In this approach the vertex (14) leads to the $NN\pi\pi$ coupling. Therefore, the $NN\pi\pi$ coupling is a necessary vertex and is resulted in that the pion is a Goldstone boson.

Replacing the quark vertex of one pion (3) in Eq. (4) by the vertex of two pions (14), the form factor and coupling constant of the the nucleon - two pions vertex are determined as

$$\mathcal{L}_{NN2\pi} = F_{NN2\pi}(q^2)\frac{1}{f_\pi}\bar{N}N\pi^2, \quad (15)$$

$$F_{NN2\pi}(q^2) = \frac{6m}{f_\pi}D_2(q^2)\left\{\frac{1}{a} + a - 1 - \frac{a q^2}{4m_N^2}\right\}, \quad (16)$$

where q is the total momenta of the two pions and $q^2 = (p' - p)^2$. The coupling constant is defined as

$$g_{NN2\pi} = F_{NN2\pi}(0) = \frac{6m}{f_\pi}\left(\frac{1}{a} + a - 1\right) = 29.13. \quad (17)$$

It is worth to point out that in this approach there is no contact interaction like

$$\bar{N}\vec{\tau}\gamma_\mu N \cdot \vec{\pi} \times \partial_\mu \vec{\pi}$$

introduced in Refs. [2,3]. However, in this approach this term can be obtained from the two couplings: the nucleon is coupled to the ρ meson and the ρ couples to two pions and it is not a contact term. In this paper only contact interactions between nucleon and pions are studied. On the other hand, the value of the coupling constant $\frac{1}{f_\pi}g_{NN2\pi}$ is much greater than the corresponding constant presented in Refs. [2,3].

In the same way the coupling constants and the form factors between nucleon and multi-pions, $NN(n\pi)$ with $n > 2$ can be predicted too. Taking the pion into account only, the interactions between quarks and pions are expressed as

$$\begin{aligned} -m\bar{\psi}e^{i\gamma_5\pi}\psi &= -im\frac{2}{f_\pi}\bar{\psi}\gamma_5\pi\psi - i\frac{2m}{f_\pi}\bar{\psi}\gamma_5\tau^i\psi\pi^i \sum_{n=1}^{\infty} \frac{1}{(2n+1)!!}(-1)^n\left(\frac{4}{f_\pi^2}\pi^2\right)^n \\ &\quad + \bar{\psi}\psi\frac{4m}{f_\pi^2}\pi^2 \sum_{n=1}^{\infty} \frac{1}{(2n)!!}(-1)^n\left(\frac{4}{f_\pi^2}\pi^2\right)^{n-1}. \end{aligned} \quad (18)$$

As studied above, the shifting of the a_μ field is another source of the pion field. Using the couplings between quarks and odd number of pions with $n \geq 3$ and the procedure showing above, the couplings between nucleon and pions with odd number are determined as

$$\mathcal{L}_{odd\pi} = F_{odd\pi}(q^2)i\bar{N}\gamma_5\tau^i\pi^iN \sum_{n=1}^{\infty} \frac{1}{(2n+1)!!}(-1)^n\left(\frac{4}{f_\pi^2}\pi^2\right)^n, \quad (19)$$

$$F_{odd\pi}(q^2) = g_{odd}D_2(q^2)\left\{1 - \frac{5}{6}\frac{ma}{f_\pi}\frac{1}{g_{odd}}\frac{q^2}{m_N^2}\right\},$$

$$g_{odd} = \frac{2m}{f_\pi}\left(\frac{1}{a} + \frac{5}{3}a - 1\right) = 17.53,$$

$$F_{odd\pi}(q^2) = g_{odd}D_2(q^2)\{1 - 0.317q^2\}, \quad (20)$$

where q is the total momenta of the pions.

Using the couplings between quarks and pions with even numbers (≥ 2) (18) and the transit matrix element (4) the couplings between nucleon and pions with even numbers are determined

$$\mathcal{L}_{even\ \pi} = F_{even\ \pi}(q^2) \frac{2}{f_\pi} \pi^2 \bar{N} N \sum_{n=1}^{\infty} \frac{1}{(2n)!!} (-1)^n \left(\frac{4}{f_\pi^2} \pi^2 \right)^{n-1}, \quad (21)$$

where $F_{even\ \pi}(q^2)$ is the same as $F_{NN2\pi}(q^2)$ (16) and $g_{even} = F_{even\ \pi}(0) = g_{NN2\pi}$ (17). The form factor is expressed as

$$F_{even\ \pi}(q^2) = g_{even} D_2(q^2) \{1 - 0.343q^2\}. \quad (22)$$

It is straight forward to extend the study above to the cases of three flavors. The chiral field theory of mesons of three flavors have been proposed [13] and the physics of the axial-vector, the vector, and the pseudoscalar mesons of three flavors are studied. Theory agrees with data well. The flavor wave functions of baryons can be found in Ref. [5]. In the chiral limit, the couplings between baryons and all the pseudoscalar mesons (pion, kaon, η , and η') can be determined without new parameter. Using the Lagrangian (1), in the same way the couplings between the baryons and the vector and the axial-vector mesons can be determined too. The detailed study is beyond the scope of this paper.

In summary, in this paper a quark model of baryon and a chiral field theory are applied to study the couplings between nucleon and pions. The coupling constant $g_{NN\pi}$ predicted is consistent with current value and the nature of Goldstone boson of pion leads to the existence

of the $NN2\pi$. It predicted the existence of the couplings of nucleon and multi-pions. The predictions made in this paper can be tested experimentally.

References

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